**PROBABILITY**: Homework 

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# Problem 1

Denote  as car in door #1, 2, 3; and E as host opens door #2



where



In order to make ,

we need 

So that, 

Therefore,



# Problem 2

## (a)

*E* and *F* are independent events. Because owing a car will not influence whether the name listed in telephone book and neither will list in book influence owing a car.

## (b)

*E* and *F* are not independent events. Because people’s height and weight shows linear relationship to some extent.

## (c)

*E* and *F* are not independent events. Because the United States is in the western hemisphere, which means E and F will happen on the same time.

## (d)

*E* and *F* are not independent events. Because the weather of two adjacent days shows somewhat relationship. And we cannot say they are irrelevant.

# Problem 3

## (a)

In order to stop the game in 4th game, the only possible case is one win 1 game and the other win 3 games. And the one win 1 game must win in round 1st or 2nd.



## (b)

Denote *A, B* as the games player A, B win. Since *A*-*B*=2, *A+B* must be an even number. Denote *P*(*n*) as the probability that player A wins in n games, and n must be even.

When A wins in n games, the  game winner must be A. And in  game, . Supposing A wins in  games, then at the  game winner must be B, and at ,  game winner must be A. So:



Which means 



which is



# Problem 4

Obviously, *X* can only be 1, 2, 3, 4, 5, 6.















# Problem 5





# Problem 6

## (a)



## (b)



# Problem 7

## (a)



## (b)

Let player wins, then they must play  times.



# Problem 8

## (a)



So *c* should be 3.

## (b)

When ,



When , 

When , 

## (c)



# Problem 9

## (a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | Y |
| 1 | .10 | .05 | .02 | .02 | **.19** |
| 2 | .05 | .20 | .05 | .02 | **.32** |
| 3 | .02 | .05 | .20 | .04 | **.31** |
| 4 | .02 | .02 | .04 | .10 | **.18** |
| X | **.19** | **.32** | **.31** | **.18** |  |

## (b)





Therefore, *X* and *Y* are not independent.

## (c)





# Problem 10

Since the point is chosen uniformly, we can get the joint pdf easily:



In order to get the value of *C*, we need to integral



This integral is equal to *C* times the area of ellipse, which is . Thus, 

Now we compute the marginal density of *X*:



Then we compute the marginal density of *Y*:



# Problem 11

## (a)

First, we can get the joint pdf is



Then we compute the marginal density of *X*:



And now we compute the marginal density of *Y*:



Since , *X* and *Y* are independent.

## (b)

In the previous part we have already get that







# Problem 12

## (a)



So 

## (b)

First, we compute the marginal density of *X*:



Then now we compute the marginal density of *Y*:



Since , *X* and *Y* are not independent.

## (c)

From the previous part, we already computed the marginal densities:





## (d)

Given *y*:



Given *x*:



# Problem 13

 is a monotone transformation. . Then



# Problem 14



Which means *Y* has cdf *F*.

# Problem 15

## (a)

Suppose that , where c is a constant number.

Then we have



Thus, 



## (b)



# Problem 16

## (a)



Thus, 

## (b)



Thus, 

# Problem 17

## (a)





## (b)



## (c)



When 

When 

## (d)

From part (a), we already computed the joint p.d.f:



## (e)

